No. of Question Paper : 6U-4:6123

[...]ique Paper Code : 2341504

[...]me of the Paper : Mathematical Physics - II , Allied Course F-9

[...]me of the Course : ~~B. Tech.~~ (Computer Science) (~~FYUP Scheme~~)

[...]mester : V

[...]ration : 3 Hours

Maximum Marks : 75

(Write your Roll no. on the top immediately on receipt of this question paper)

Do *five* questions in all.

Question No. 1 is compulsory.

Do any *five* questions : (3 × 5 = 15)

(a) Calculate the value of Wronskian for:

$e^x \sin x$ and $e^x \cos x$

(b) Determine the order, degree and linearity of the differential equation:

$$\left(\frac{d^3y}{dx^3}\right)^2 + 5\left(\frac{d^2y}{dx^2}\right)^4 + 9\left(\frac{dy}{dx}\right)^5 + 7y = 0$$

(c) Prove the following property of Poisson Bracket:

$$[2u+3v, w] = 2[u, w] + 3[v, w]$$

(d) Write the Euler-Lagrange's equation for the function $f(x, y, y')$.

(e) Form the differential equation whose solution is:

$Ae^{2x} + Be^{3x}$

(f) Prove that: $\frac{1}{D-a} f(x) = e^{ax} \int f(x) e^{-ax} dx$

(g) Write the Lagrangian and Hamiltonian of a free particle.

(h) Solve :

$$(y \sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$$

Solve the following differential equations:

(a) $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^2 \cos^2 y$ (6)

(b) $\frac{dy}{dx} = \frac{2x-5y+3}{4y+2x-6}$ (9)

3. Solve the following differen[illegible] equations:

(a) $\left(D^4 - 4a^2 D^2\right) y = \cos bx$ (6)

(b) $\dfrac{d^2 y}{d x^2} - 6 \dfrac{d y}{d x} + 9 y = 2 x e^{3x} \sin 4x$ (9)

4. (a) Solve the following differential equation

$$\left(y^3 - 2 y x^2\right) d x + \left(2 x y^2 - x^3\right) d y = 0$$ (6)

(b) Using the method of variation of parameters, solve

$$\left(x^2 D^2 - 2 x D + 2\right) y = x \log x \ ; \ D \equiv \frac{d}{d x}$$ (9)

5. (a) Using the method of undetermined coefficients, solve

$$\frac{d^2 y}{d x^2} - 5 \frac{d y}{d x} + 6 y = e^{2x} + e^{3x}$$ (7)

(b) Solve the coupled differential equations:

$$\frac{d x}{d t} = 2 x - 3 y$$

$$\frac{d y}{d t} = y - 2 x$$ (8)

6. (a) Prove that the equation of the shortest path between two points on surface of right circular cylinder of radius a is given by

$$z = c_1 \phi + c_2 ,$$

where c_1 and c_2 are constants. (6

(b) Using Lagrange's method of undetermined multiplier, find the m imum value of u = (sin x) (sin y) (sin z) where the variables x, y, z the angles of a triangle. (9

(a) Given F = $q_1^2 + q_2^2$ and G = $2p_1 + p_2$, where q_1 and q_2 are generali coordinates and p_1 and p_2 are the corresponding generalized mom Prove that (9)

$$[F, G] = 4q_1 + 2q_2$$

(b) Show that

(i) $\{ q_k , H\} = \frac{1}{\ldots}$

(ii) $\{ p_k , H\} = \dfrac{1}{\dot{p}_k}$ (3, 3)

here, H denotes Hamiltonian and $1 \le k \le n$

(a) Write the Lagrangian of a simple harmonic oscillator (mass m and force constant k) and hence obtain its equation of motion. (2, 4)

(b) Show that the path followed by a particle in a vertical plane in sliding from one point to another in the absence of friction in the shortest time is cycloid. (9)